# Design of Synchronous Motor Using Advanced Inverse Cosine Function to Reduce Harmonics of Induced Voltage Under Load Condition

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The harmonics of the induced voltage under load conditions can cause the deterioration of the motor performance. It occurs when the peak value of the demand voltage exceeds the DC link voltage. Thus, to analysis and reduce the induced voltage harmonics under load conditions, the induced voltage is separated into three components. Through the separation, the induced voltage of the concentrated flux synchronous motor (CFSM), the reference motor, is analyzed. Consequently, the dominant component causing the voltage harmonics is determined. In addition, based on the results, the design method using the advanced inverse cosine function (AICF) is proposed in this paper. Using this method, an asymmetric rotor shape is determined by considering the armature reaction. As a result, an improved CFSM achieving the required performance is designed. Lastly, the induced voltages and input currents of the motors are compared through finite-element analysis (FEA) as well as experiments.

Index Terms—Advanced inverse cosine function (AICF), concentrated flux synchronous motor (CFSM), induced voltage, permanent magnet synchronous motor, voltage harmonics

## I. INTRODUCTION

MANY types of electrical motors are applied to industrial equipment, medical equipment, home appliances, automobile parts, robots, etc. Especially, they have become essential components of vehicles as many automobile parts are applying electrical equipment. The harmonics of the induced voltage, however, has become a new issue as the usage of synchronous motors has increased in the automobile field [1]. This is because the motors for automotive applications have a high power density due to their space and weight limitations. In addition, this issue is becoming increasingly important because the motors of automobiles should be able to operate under the voltage limitation due to the battery usage in vehicles. If the peak value of the induced voltage under load conditions is partially higher than the DC link voltage as shown in Fig. 1, the inverter cannot apply the demand magnitude or waveform of the current to the armature coil. Therefore, the actual performance of the motor will decline compared to the predicted performance [2]-[4]. This is because only the fundamental harmonic is considered for the d, q-axis equivalent circuit of the electric motors, which is generally used to predict the performance of the motors. Consequently, if the harmonic components of the induced voltage are not considered in the design method, the required speed or torque performance cannot be achieved.

In this paper, the induced voltage under load conditions is analytically separated into three components based on the voltage equations of electric motors to determine the dominant cause. Considering the deominant cause affecting the harmonics, a design method of the synchronous motor using the advanced inverse cosine function (AICF) is proposed. This is an design method that can achieve sinusoidal flux linkage and inductance waveforms at the air gap under a specific load condition. In the full paper, a reference motor will be proposed and its voltage harmonics will be analyzed. Based on the

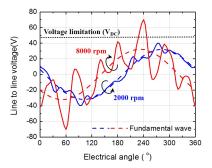


Fig. 1. Induced voltages at 2000 and 8000 rpm under the same power

results, the improved motor with an asymmetric rotor shape using AICF will be designed. Finally, the induced voltage of the improved motor will be compared with that of the reference motor to verify the analysis and design methods.

### II. ANALYSIS OF THE INDUCED VOLTAGE HARMONICS

A mathematical process is presented herein to separate the induced voltage. The phase variable mathematical models can be depicted by three phases in synchronous motors. The voltage equation consists of phase resistance R, input current  $i_x$ , and linkage flux of a phase  $\lambda_x$  [5]. x and y are the phases, such as a, b, and c. The end-coil inductance is neglected. The linkage flux is composed of the self-inductance  $L_{xx}$ , mutual inductance  $L_{xy}$ , input current, and linkage flux by the permanent magnet (PM)  $\lambda_m$ . Thus, the derivative for each linkage flux can be expressed as follows:

$$\begin{vmatrix} \frac{d\lambda_a}{dt} = L_{aa} \frac{di_a}{dt} + L_{ab} \frac{di_b}{dt} + L_{ac} \frac{di_c}{dt} + \omega \bigg[ i_a \frac{dL_{aa}}{d\theta} + i_b \frac{dL_{ab}}{d\theta} + i_c \frac{dL_{ac}}{d\theta} \bigg] + \omega \frac{d\lambda_m}{d\theta} \\ \frac{d\lambda_b}{dt} = L_{ba} \frac{di_a}{dt} + L_{bb} \frac{di_b}{dt} + L_{bc} \frac{di_c}{dt} + \omega \bigg[ i_a \frac{dL_{ba}}{d\theta} + i_b \frac{dL_{bb}}{d\theta} + i_c \frac{dL_{bc}}{d\theta} \bigg] + \omega \frac{d\lambda_m}{d\theta} \\ \frac{d\lambda_c}{dt} = L_{ca} \frac{di_a}{dt} + L_{cb} \frac{di_b}{dt} + L_{cc} \frac{di_c}{dt} + \omega \bigg[ i_a \frac{dL_{ca}}{d\theta} + i_b \frac{dL_{cb}}{d\theta} + i_c \frac{dL_{cc}}{d\theta} \bigg] + \omega \frac{d\lambda_m}{d\theta}$$
(1)

L as the discrete values of its waveforms according to i and

 $\theta$  can be obtained from nonlinear finite-element analysis (FEA). Given the derivatives of the phase linkage flux, the first to third terms represent the differential flux linkage components in the winding due to the varying current, the fourth to sixth terms shows the differential flux linkage components in the winding due to the inductance change, and the last terms present the variation rate of the linkage flux by rotating PM. In the full paper, the voltage harmonics of the reference motor are analyzed based on (1). Through the analytical method, the induced voltage waveform of the reference motor will be separated into three components. Then the components will be analyzed according to the load conditions. The results will help determine the dominant cause of the voltage harmonics.

# III. DESIGN METHOD USING ADVANCED INVERSE COSINE FUNCTION (AICF)

AICF is an equation that adjusts the air gap length by considering the effect of the armature reaction. For the convenience of the analytical approach, the permeability of the magnetic core is assumed to be infinite. In addition, it is assumed that the magnetomotive force (MMF) distribution by the armature winding can be considered sinusoidal because only the fundamental component of the armature MMF is considered in this study, regardless of the winding method. Fig. 2 is a simplified representation of the air gap MMF considering the effect of the armature reaction under load conditions. In Fig. 2, the red line with the square symbol as the resultant air gap MMF is the sum of the MMFs by the PM and armature winding. Consequently, the armature reaction causes a decrease in the resultant air gap flux density under one half of the pole, and an increase under the other half. AICF enlarges the air gap length of the area where the air gap flux density is increased, and reduces the air gap length of the area where the flux density is reduced by the armature reaction. Therefore, the air gap flux density distribution is rendered sinusoidal, as shown in Fig. 3. Equation (2) shows the advanced inverse cosine function to determine the air-gap length according to  $\theta$ .

$$l_g(\theta) = \frac{l_{\min}}{\cos\theta} - k_F \frac{l_{\min}\sin(\theta + \beta)}{\cos\theta}$$
(2)

where  $l_g(\theta)$  is the air gap length by  $\theta$ , and  $\theta$  varies from -90 to 90° in the electrical angle.  $k_F$  is the ratio of the air gap MMFs by the PM and the armature current.  $\beta$  is the current phase angle.  $l_{\min}$ , is the minimum value of the air gap length. In the full paper, detail explanations of the AICF will be described. In addition, the improved motor will be design by the proposed method, and the voltage harmonics of the improved and reference motors will be compared.

## IV. CONCLUSION AND FUTURE WORK

The harmonics of the induced voltage can cause the performance of the electric motors to deteriorate, as pointed out in the introduction. Therefore, in this study, the induced voltage of the reference motor is analyzed by separating the voltage into its three components using the proposed method.

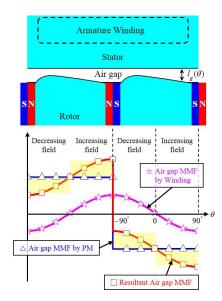


Fig. 2. Air gap MMF distribution considering armature reaction



Fig. 3. Air gap MMF and flux density considering armature reaction

From the separated voltage waveforms, the dominant component causing the induced voltage harmonics will be determined. Besides, a design method using the AICF is proposed, and an improved motor will be designed in the full paper. Lastly, the validity of the proposed design method will be verified by comparing the induced voltage harmonics obtained from simulations and experiments.

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